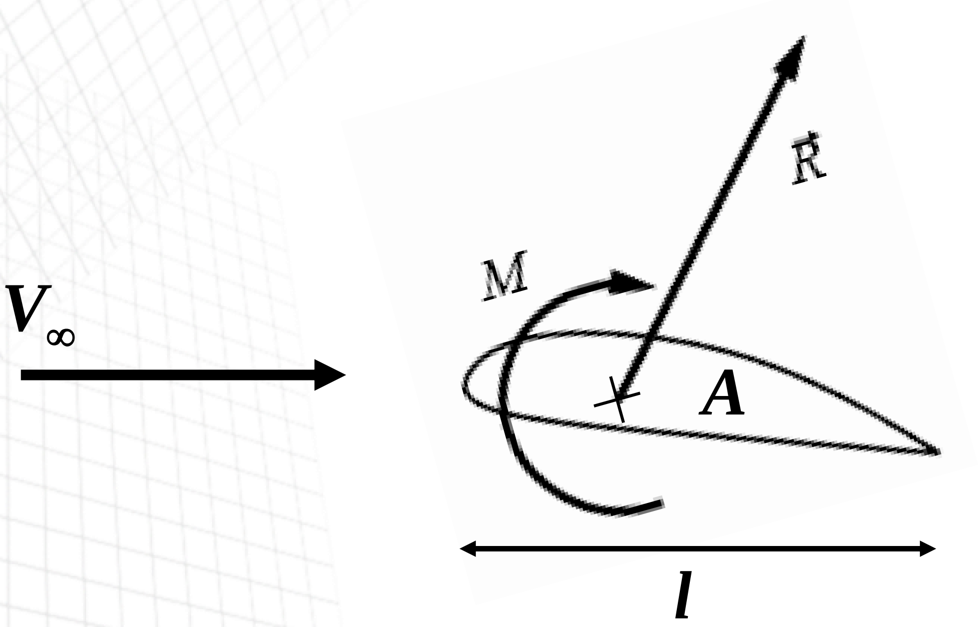
# < 1.7. Dimensional analysis >

## ❖ Physical parameters

- Q: What physical quantities influence forces and moments?



# < 1.7. Dimensional analysis >

## ❖ Physical parameters

- Physical quantities to be considered

| Parameter | Symbol | units |
|---|---|---|
| Lift | $L'$ | $MLT^{-2}$ |
| Angle of attack | $\alpha$ | - |
| Freestream velocity | $\mathbf{V}_\infty$ | $LT^{-1}$ |
| Freestream density | $\rho_\infty$ | $ML^{-3}$ |
| Freestream viscosity | $\mu_\infty$ | $ML^{-1}T^{-1}$ |
| Freestream speed of sound | $a_\infty$ | $LT^{-1}$ |
| Size of body (e.g. chord) | $c$ | L |

- Generally, resultant aerodynamic force:

$$R = f(\rho_\infty, \mathbf{V}_\infty, c, \mu_\infty, a_\infty) \qquad (1)$$

# < 1.7. Dimensional analysis >

## ❖ The Buckingham PI Theorem

- The relation with N physical variables

$$f_1 ( p_1 , \; p_2 , \; p_3 , \; \ldots , p_N ) = 0$$

can be expressed as

$$f_2 ( \Pi_1 , \; \Pi_2 , \; \ldots , \Pi_{N-K} ) = 0$$

where K is the No. of fundamental dimensions

- *Then* $\Pi_1 = f_3 ( p_1 , \; p_2 , \; \ldots , p_K , p_{K+1} )$

$$\Pi_2 = f_4 ( p_1 , \; p_2 , \; \ldots , p_K , p_{K+2} )$$

....

$$\Pi_{N-K} = f ( p_1 , \; p_2 , \; \ldots , p_K , p_N )$$

## < 1.7. Dimensional analysis >

## ❖ The Buckingham PI Theorem

**Example)**

$$f(R, \rho_\infty, v_\infty, c, \mu_\infty, a_\infty) = 0$$

Buckingham's $\Pi$ theorem, fundamental dimensions are

$$\begin{aligned} m &= mass \\ l &= length \quad \Rightarrow k = 3 \\ t &= time \end{aligned}$$

# < 1.7. Dimensional analysis >

## ❖ The Buckingham PI Theorem

### Example)

Now each physical variable can be expressed by fundamental dimensions

$$
\begin{aligned}
&[R] = mlt^{-2} \\
&[\rho_\infty] = ml^{-3} \\
&[v_\infty] = lt^{-3} \\
&[c] = l \qquad\qquad \Rightarrow \quad N = 6 \\
&[\mu_\infty] = ml^{-1}t^{-1} \\
&[a_\infty] = lt^{-1}
\end{aligned}
$$

Therefore $f_2(\pi_1, \pi_2, \pi_3) = 0$

Set,

$$
\begin{aligned}
\pi_1 &= f_3(\rho_\infty, v_\infty, c, R) \\
\pi_2 &= f_4(\rho_\infty, v_\infty, c, \mu_\infty) \\
\pi_3 &= f_5(\rho_\infty, v_\infty, c, a_\infty)
\end{aligned}
$$

# < 1.7. Dimensional analysis >

## ❖ The Buckingham PI Theorem

ex) $$\pi_1 = \rho_\infty^\alpha v_\infty^\beta c^\gamma R$$

$$[\pi_1] = (ml^{-3})^\alpha (lt^{-1})^\beta (l)^\gamma (mlt^{-2})$$

For non-dimensionalization

$$m : \alpha + 1 = 0$$

$$l : -3\alpha + \beta + \gamma + 1 = 0 \quad \Rightarrow \begin{cases} \alpha = -1 \\ \beta = -2 \\ \gamma = -2 \end{cases}$$

$$t : -\beta - 2 = 0$$

$$\therefore \pi_1 = R\rho_\infty^{-1} v_\infty^{-2} c^{-2} = \frac{R}{\rho_\infty v_\infty^2 c^2}$$

< 1.7. Dimensional analysis >

## ❖ The Buckingham PI Theorem

Through similar procedure

$$\pi_2 = \frac{\rho_\infty v_\infty c}{\mu_\infty}$$

$$\pi_3 = \frac{v_\infty}{a_\infty}$$

In summary,

$$f_2\left(\frac{R}{\rho_\infty v_\infty^2 c^2}, \frac{\rho_\infty v_\infty c}{\mu_\infty}, \frac{v_\infty}{a_\infty}\right) = 0$$

# < 1.7. Dimensional analysis >

## ❖ Dimensionless form

Define ; 1. Dimensionless force coefficient $C_R = \dfrac{R}{(\frac{1}{2}\rho_\infty v_\infty^2 S)}$

2. Reynolds number $Re = \dfrac{\rho_\infty v_\infty c}{\mu_\infty}$

3. Mach number $M = \dfrac{v_\infty}{a_\infty}$

$\therefore C_R = f(Re, M)$

# < 1.8. Flow similarity >

## ❖ Dynamic similarity

- Two different flows are **dynamically similar** if
  - Streamline patterns are similar
  - Velocity, pressure, temperature distributions are same
  - Force coefficients are same

- Criteria
  - Geometrically similar
  - Similarity parameters (Re, M) are same

# < 1.8. Flow similarity >

## ❖ Dynamic similarity

- Airfoil flow example
  - Consider two airfoils which have the same shape and angle of attack, but have **<u>different sizes</u>** and are operating in **<u>two different fluids</u>**.

# < 1.8. Flow similarity >

## ❖ Dynamic similarity

| Airfoil 1 (sea level) | Airfoil 2 (cryogenic tunnel) |
|---|---|
| $\alpha_1=5°$ | $\alpha_2=5°$ |
| $V_1=210$ m/s | $V_2=140$ m/s |
| $\rho_1=1.2$ kg/m$^3$ | $\rho_2=3.0$ kg/m$^3$ |
| $\mu_1=1.8e\text{-}5$ kg/m·s | $\mu_2=1.5e\text{-}5$ kg/m·s |
| $a_1=300$ m/s | $a_2=200$ m/s |
| $c_1=1.0$ m | $c_2=0.5$ m |

Airfoil 1 – Sea level air

Airfoil 2 – Cryogenic tunnel

# < 1.8. Flow similarity >

## ❖ Dynamic similarity

- The PI products evaluate to the following values.

| Airfoil 1 (sea level) | Airfoil 2 (cryogenic tunnel) |
|---|---|
| $\alpha_1=5°$ | $\alpha_2=5°$ |
| $RE_1=1.4e7$ | $RE_2=1.4e7$ |
| $M_1=0.7$ | $M_2=0.7$ |

- Two airfoil flows are dynamically similar.

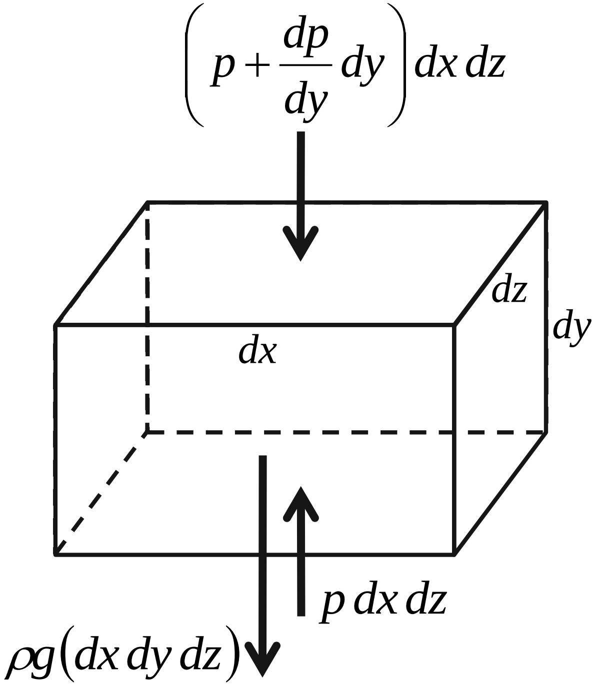
$$g(\alpha_1, \mathrm{Re}_1, M_1) = g(\alpha_2, \mathrm{Re}_2, M_2)$$

$$c_{l_1} = c_{l_2}$$

# < 1.9. Fluid statics : Buoyancy force >

## ❖ Fluid statics

- The force on an element of fluid itself
  - *Pressure forces* from the surrounding fluid exerted on the surface on the element
  - *Gravity force* due to the weight of the fluid inside the element



# < 1.9. Fluid statics : Buoyancy force >

## ❖ Fluid statics

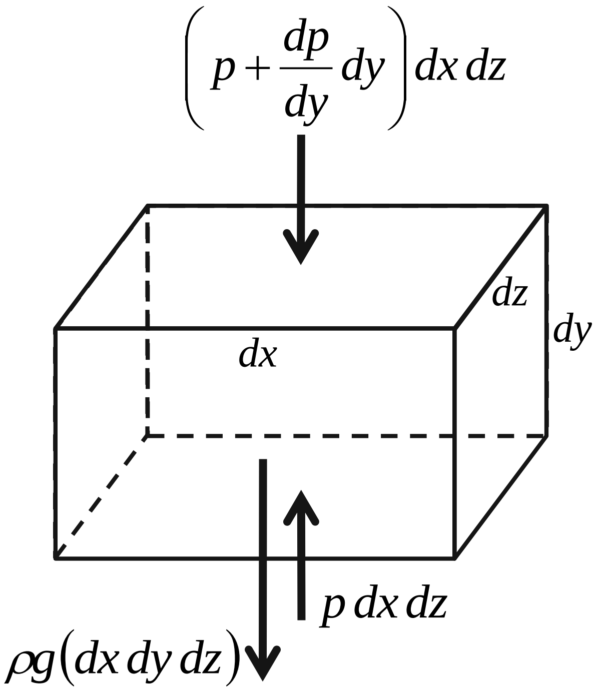
- Net pressure force & Gravity force

$$(Net\ pressure\ force)$$

$$= p(dx\,dz) - \left(p + \frac{dp}{dy}dy\right)(dx\,dz)$$

$$= -\frac{dp}{dy}(dx\,dy\,dz)$$

$$(Gravity\ force) = -\rho(dx\,dy\,dz)g$$



# < 1.9. Fluid statics : Buoyancy force >

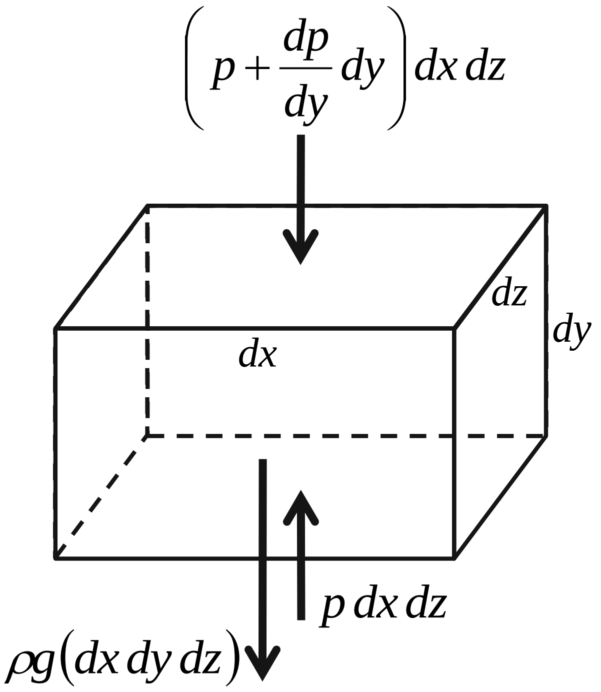
## ❖ Fluid statics

- The fluid element is in equilibrium,

$$-\frac{dp}{dy}(dx\,dy\,dz)-g\rho(dx\,dy\,dz)=0$$

$$dp=-g\rho\,dy$$

→ *Hydrostatic equation*



# < 1.9. Fluid statics : Buoyancy force >

## ❖ Fluid statics

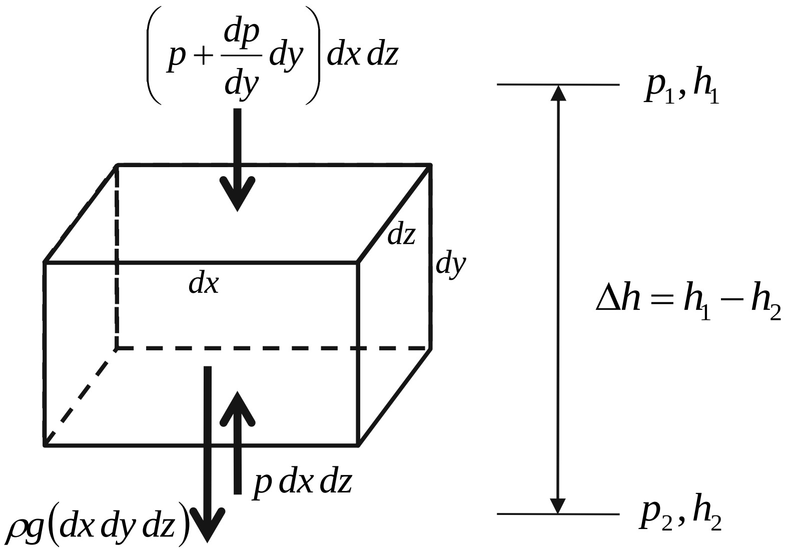
- Let the fluid be a liquid, for which we can assume $\rho$ is constant.

$$\int_{p_1}^{p_2} dp = -\rho g \int_{h_1}^{h_2} dy$$

$$p_2 - p_1 = -\rho g \left(h_2 - h_1\right)$$

$$p_1 + \rho g h_1 = p_2 + \rho g h_2$$

$$p + \rho g h = const.$$

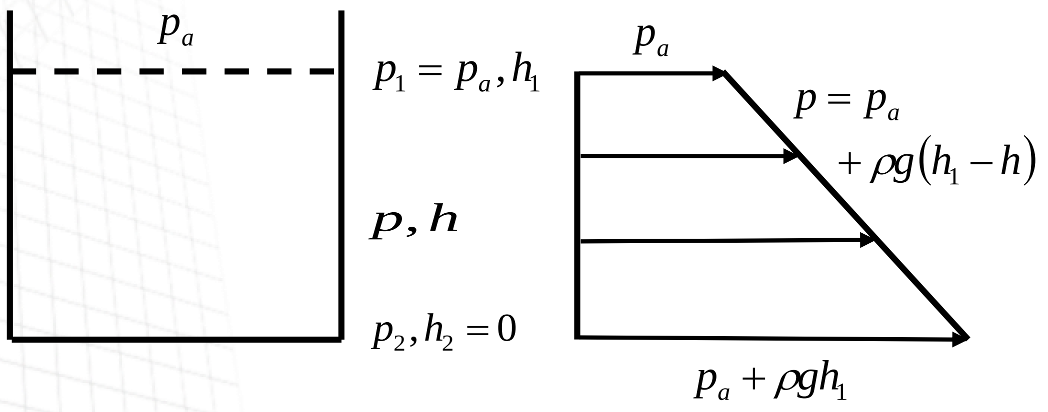


# < 1.9. Fluid statics : Buoyancy force >

## ❖ Fluid statics

- Applications : Walls of container

$$p + \rho g h = p_1 + \rho g h_1 = p_a + \rho g h_1$$
$$p = p_a + \rho g \left(h_1 - h\right)$$



# < 1.9. Fluid statics : Buoyancy force >

## ❖ Fluid statics

- Applications : U-tube manometer

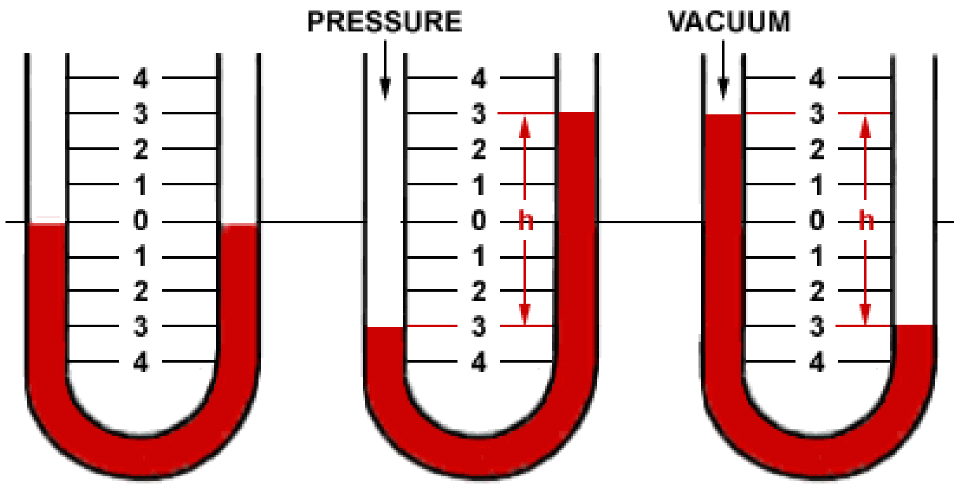



Fig. 2-1 Fig. 2-2 Fig. 2-3

# < 1.9. Fluid statics : Buoyancy force >

## ❖ Buoyancy force

- Archimedes principle

| Buoyancy force on body | = | Weight of fluid displaced by body |
|---|---|---|